



U.M.A

Commission OPAM



OPAM 2016  
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## 24<sup>th</sup> PAN AFRICAN MATHEMATICS OLYMPIAD

Day 2 : Thursday, April 28, 2016

Duration : 4 h 30 min

### PROBLEM 1

Let  $x, y, z$  be positive real numbers such that  $xyz = 1$ . Prove that

$$\frac{1}{(x+1)^2 + y^2 + 1} + \frac{1}{(y+1)^2 + z^2 + 1} + \frac{1}{(z+1)^2 + x^2 + 1} \leq \frac{1}{2}.$$

### PROBLEM 2

Let  $ABCD$  be a trapezium such that the sides  $AB$  and  $CD$  are parallel and the side  $AB$  is longer than the side  $CD$ . Let  $M$  and  $N$  be on the segments  $AB$  and  $BC$  respectively, such that each of the segments  $CM$  and  $AN$  divides the trapezium in two parts of equal area.

Prove that the segment  $MN$  intersects the segment  $BD$  at its midpoint.

### PROBLEM 3

Consider an  $n \times n$  grid formed by  $n^2$  unit squares. We define the center of a unit square as the intersection of its diagonals.

Find the smallest integer  $m$  such that, choosing any  $m$  unit squares in the grid, we always get four unit squares among them whose centers are vertices of a parallelogram.