



U.M.A

Commission OPAM



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## 24<sup>th</sup> PAN AFRICAN MATHEMATICS OLYMPIAD

Day 1 : Wednesday, April 27, 2016

Duration : 4 h 30 min

### PROBLEM 1

Two circles  $\mathcal{C}_1$  and  $\mathcal{C}_2$  intersect each other at two distinct points  $M$  and  $N$ . A common tangent line touches  $\mathcal{C}_1$  at  $P$  and  $\mathcal{C}_2$  at  $Q$ , the line being closer to  $N$  than to  $M$ . The line  $PN$  meets the circle  $\mathcal{C}_2$  again at the point  $R$ .

Prove that the line  $MQ$  is a bisector of the angle  $\angle PMR$ .

### PROBLEM 2

We have a pile of 2016 cards and a hat. We take out one card, put it in the hat and then divide the remaining cards into two arbitrary non empty piles. In the next step, we choose one of the two piles, we move one card from this pile to the hat and then divide this pile into two arbitrary non empty piles.

This procedure is repeated several times : in the  $k$ -th step ( $k > 1$ ) we move one card from one of the piles existing after the step  $(k - 1)$  to the hat and then divide this pile into two non empty piles.

Is it possible that after some number of steps we get all piles containing three cards each ?

### PROBLEM 3

For any positive integer  $n$ , we define the integer  $P(n)$  by :

$$P(n) = n(n + 1)(2n + 1)(3n + 1)\dots(16n + 1).$$

Find the greatest common divisor of the integers  $P(1), P(2), P(3), \dots, P(2016)$ .