

23<sup>rd</sup> edition of the Pan African Mathematics Olympiad

Abuja: 20 August - 29 August, 2015

First Day: 24 August 2015

Duration : 4 h 30

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1. Prove that

$$\sqrt{x-1} + \sqrt{2x+9} + \sqrt{19-3x} < 9$$

for all real  $x$  for which the left-hand side is well defined.

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2. A convex hexagon  $ABCDEF$  is such that

$$AB = BC \quad CD = DE \quad EF = FA$$

and

$$\angle ABC = 2\angle AEC \quad \angle CDE = 2\angle CAE \quad \angle EFA = 2\angle ACE.$$

Show that  $AD$ ,  $CF$  and  $EB$  are concurrent.

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3. Let  $a_1, a_2, \dots, a_{11}$  be integers. Prove that there are numbers  $b_1, b_2, \dots, b_{11}$ , each  $b_i$  equal  $-1, 0$  or  $1$ , but not all being  $0$ , such that the number

$$N = a_1b_1 + a_2b_2 + \dots + a_{11}b_{11}$$

is divisible by 2015.

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