



PAMO 2019 Day 1

4 April 2019

Duration: 4 h 30 min

1. Let $(a_n)_{n=0}^{\infty}$ be a sequence of real numbers defined as follows:

- $a_0 = 3$, $a_1 = 2$, and $a_2 = 12$; and
- $2a_{n+3} - a_{n+2} - 8a_{n+1} + 4a_n = 0$ for $n \geq 0$.

Show that a_n is always a strictly positive integer.

(7 points)

2. Let k be a positive integer. Consider k not necessarily distinct prime numbers such that their product is ten times their sum. What are these primes and what is the value of k ?

(7 points)

3. Let ABC be a triangle, and D, E, F points on the segments BC, CA, AB respectively such that

$$\frac{BD}{DC} = \frac{CE}{EA} = \frac{AF}{FB}.$$

Show that if the centres of the circumscribed circles of the triangles DEF and ABC coincide, then ABC is an equilateral triangle.

(7 points)

La version française se trouve de l'autre côté de la page.

(Total: 21 points)