



U.M.A

Commission OPAM



## 25<sup>th</sup> PAN AFRICAN MATHEMATICS OLYMPIAD

Rabat from 1 to 7 july 2017

Day 2 : Wednesday, july 5, 2017

Duration : 4 h 30 min

### PROBLEM 4

Find all the real numbers  $x$  such that  $\frac{1}{[x]} + \frac{1}{[2x]} = \{x\} + \frac{1}{3}$ , where  $[x]$  denotes the integer part of  $x$  and  $\{x\} = x - [x]$ .  
For example,  $[2.5] = 2$ ,  $\{2.5\} = 0.5$  and  $[-1.7] = -2$ ,  $\{-1.7\} = 0.3$ .

### PROBLEM 5

The numbers from 1 to 2017 are written on a board. Deka and Farid play the following game : each of them, on his turn, erases one of the numbers. Anyone who erases a multiple of 2, 3 or 5 loses and the game is over. Is there a winning strategy for Deka ?

### PROBLEM 6

Let  $ABC$  be a triangle with  $H$  its orthocenter. The circle with diameter  $[AC]$  cuts the circumcircle of the triangle  $ABH$  at  $K$ . Prove that the point of intersection of the lines  $CK$  and  $BH$  is the midpoint of the segment  $[BH]$ .