



U.M.A

Commission OPAM



25th PAN AFRICAN MATHEMATICS OLYMPIAD

Rabat from 1 to 7 july 2017

Day 1 : Tuesday, july 4, 2017

Duration : 4 h 30 min

PROBLEM 1

We consider the real sequence (x_n) defined by $x_0 = 0$, $x_1 = 1$ and $x_{n+2} = 3x_{n+1} - 2x_n$ for $n = 0, 1, \dots$

We define the sequence (y_n) by $y_n = x_n^2 + 2^{n+2}$ for every nonnegative integer n .
Prove that for every $n > 0$, y_n is the square of an odd integer.

PROBLEM 2

Let x , y and z be positive real numbers such that $xy + yz + zx = 3xyz$.

Prove that $x^2y + y^2z + z^2x \geq 2(x + y + z) - 3$.

In which case do we have equality ?

PROBLEM 3

Let n be a positive integer. Find, in terms of n , the number of pairs (x, y) of positive integers that are solutions of the equation : $x^2 - y^2 = 10^2 \cdot 30^{2n}$.

Prove further that this number is never a square.